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## Appropriated Excitation
## Shock Loading Design

# Modal Analysis Using Appropriated Excitation Techniques

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**There is renewed interest in normal mode testing sparked by advances in shaker control automation and modern aircraft fabrication methods. This article covers the use of appropriated excitation of a structure to assure that only one mode shape is excited. Modes are separated physically rather than mathematically. The technique is applied to ground vibration tests of a new aircraft.**

Modal Analysis of a structure determines its natural frequencies, normal-mode shapes and generalized parameters (mass, stiffness and damping) over a specified frequency bandwidth. These characteristics are fixed for a linear structure and can be considered as an "identity card" for use in complementary calculations (stress, fatigue, dynamic behavior prediction, performance, etc . . . ).

Modal Analysis can be performed by calculation (Finite Element Modeling) or by testing. In the latter case, there are several methods for extracting the modal parameters. These can be classified in two groups:

- Global excitation
- Appropriated excitation

The global excitation method uses single or multiple broadband excitation signals to excite the structure. The intent is to excite all desired modes simultaneously. Resulting Frequency Response Function (FRF) measurements contain the required information. Curve-fitting algorithms are applied which extract the modal data from the measured FRFs. The curve-fitting algorithms separate the modes of the structure *mathematically*.

Appropriated excitation (also referred to as Normal Mode testing) takes a different philosophical approach. Here, a distribution of coherent sinusoidal forces is applied to the structure at a resonant frequency. The distribution of force amplitudes is 'tuned' to assure that only one mode shape is excited. In this process, modal separation is produced *physically* rather than mathematically. Each mode is studied in its purest possible form (in isolation from all other modes). Once a mode is physically isolated, its shape is captured from the array of measured responses and its generalized parameters are extracted from the measured mechanical power at the drive-sites and validated by inducing additional quadrature force components and observing their effect. The foundation of this technique is based upon a simple phase criterion: the mode is said to be isolated if all the velocity responses are either exactly in-phase or out-of-phase with the applied forces.

## A Short History of Modal Testing

Over the last fifty years, experimental modal parameter estimation has evolved two concepts: *physical* separation of modes using sine excitation and *mathematical* separation of modes following broadband Global excitation. The physical methods were developed first. Initial efforts focused solely on the frequency-selective nature of the resonant phenomenon and the ability of a sinusoid to predominently excite a single mode. These early *physical* methods actually used global excitation, but did not rely on mathematical processes to separate modes owing to the nature of the test equipment.

The aircraft industry need to produce *complete* Modal models of the highest accuracy gave rise to the birth of *appropriated force* Normal Mode testing which made use of the *spatial filtering* provided by the orthogonal mode shapes of a structure. This same need drove the initial development of large-scale dynamic modeling tools culminating in today's sophisticated Finite Element design programs and the supporting methods that compare (and rectify) design models with test observations.

The advent of the FFT analyzer was the impetus for a new technique and led directly to the development of *mathematical* methods to separate modes. FFT analyzers deal very efficiently with broadband signals, leading to strong interest in random and impulse modal testing. Sinusoidal testing techniques fell from vogue (most notably in the United States) in the then-emerging age of Computer Aided Testing as the sine offered no special merit when used with an FFT. Broadband excitation techniques dominated in structural testing due to the advent of affordable FFT processors, refined impulse hammers, tailored random generators, and the dispersed understanding of digital signal processing.

But, in a few aerospace organizations around the world, modal testing using *appropriated force* technology continued to evolve. This was driven by the understanding that the dynamics of a structure are physically decomposed by frequency and position. Response of a structure is dominated by an isolated mode when a driving sinusoid is tuned to near coincidence with a natural frequency and its response is enhanced by judicial selection of the forcing locations. This segregation provides a major step toward modal parameter identification prior to the application of *any* signal processing and enhances the signal/noise ratio for any subsequent analytic procedures.

The use of appropriated force testing was justified by technical developments in other quarters. The materials from which an aircraft is constructed and the manner in which it is built have changed dramatically. Aluminum skins riveted over longeron stringers and station frames have given way to composite airframes woven from multiple high-strength fibers embedded in various resins over foam and honeycomb cores. The resulting structures are stronger, lighter and exhibit vastly improved load-path redundancy. They also exhibit structural anisotropy and much higher damping levels, leading to less evident vibratory response and load-dependent nonlinearity. This changes the options available to perform a meaningful Ground Vibration Test (GVT).

Over the same course of time, finite-element modeling tools grew in capability to deal with these new materials and fabrication methods. The dependence on FEM tools increased with an attendant reduction in preliminary hardware prototyping. This placed increased importance on the design model validation provided by a full-scale GVT.

The tight integration of signal control/processing hardware with inexpensive computers of immense power provided the natural platform to perform structural measurements. The importance of matching detailed design models (now running on the same computer) to observable hardware renewed the quest for modal measurements of the highest precision and caliber from vibration-resistant modern structures. This has led to a resurgence of interest in *force appropriated* testing systems with the additional benefit that both techniques can be implemented on the same analysis platform.


*Figure 1. Appropriated-Excitation Normal Mode testing is typically applied to large and supple aerospace structures such as this complete airframe being tested by ALENIA in Italy. Such tests are serious large-scale efforts carefully conducted upon expensive structures. This examination involves 8 shakers and 896 response points.*

## Overview of Apportioned Excitation Testing

A typical force-apportioned Normal Mode test involves five steps; these are:

1. Identify all modes using a global search technique with sine sweep or random excitation.
2. Appropriate the excitation for each mode in turn. This is a complex process where the following points must be considered: a) the number and location of shakers will need to be optimized for each mode; b) the excitation frequency must be optimized with extremely high accuracy; c) the (relative) excitation levels (and phases) must be balanced (tuned); and d) the global level of the force must be set to an appropriate level.
3. Acquire the natural shape.
4. Calculate the generalized parameters.
5. Plot the impedance curves to characterize non-linearities.

Each of these test phases is discussed in ensuing paragraphs.

## Identification of Modes

The first stage of the test is to identify the structural resonances and derive an overview of the basic modes within the frequency range of interest. Sine or random excitation can be used, but sinusoidal excitation is generally preferred. Sine excitation concentrates the excitation energy at a single frequency and allows the structural response to build more than would be the case with random. Random does have the advantage that the time taken to make the complete FRF measurement across the desired frequency range is very brief, but the results are less reliable.

Using 1 or 2 shakers attached to the structure with coupling rods, (each coupling rod has a force transducer to act as the FRF reference for that point), a stepped sine sweep is used to measure the Frequency Response Functions (FRFs) at all the measurement points simultaneously. It is important during all stages of testing that the structure be protected from over test. Usually the coupling rods also act as fuses with a weak link deliberately engineered as a point of fracture in case an excess of energy is applied. Fortunately, modern Normal Mode test systems such as the Data Physics SignalStar® Matrix also incorporate limits and protection strategies that pause the test if an exceedance is sensed.

*Figure 2. Details of shaker attachments to vertical stabilizer (left) and starboard wing (right). Note the use of suspended inertial-reaction shakers driving through thin 'quill shaft' or 'stinger'rods to surface-attached force sensors. This arrangement minimizes (unmeasured) constraint moments at the drive sites and the rods act as sacrificial mechanical 'fuses' to protect the structure from unintended overload.*

Depending upon the position of the excitation shaker(s) and the modal behavior of the structure under test, it is possible that the first sine sweep may not identify all structural modes. After the first FRF measurement set is taken, the shaker(s) should be moved and another set of FRFs taken. Depending upon the complexity of the structure, more positions may need to be used to be sure of measuring all responses properly. The resulting FRFs are interpreted to roughly identify all probable modes within the specified frequency bandwidth. This information is vital for the engineer to know where the excitation shakers are to be positioned to force the structure into its pure response during the appropriation stage. Where two shakers are used, the system can be programmed for the excitation to be in phase or 180° phase shifted (in opposition).

An open loop (fixed amplitude) excitation signal can be used but it is better to employ closed loop control of both the force and phase at the excitation points. At each step frequency the structure should be allowed a stabilization time for the response to settle.

## Force Appropriation

From the identification stage, the structural resonances have been identified and characterized with an approximate mea-


*Figure 3. Typical controls and multi-Lissajous display used while tuning a mode. One 'slider' is provided for each shaker and adjusts the relative force (and phase) applied by that driver. A Global slider sets the absolute force levels by varying all drives while maintaining their relative strengths. The exact shaker frequency may be adjusted with 0.1 PPM resolution. Each velocity-versus- force Lissajous figure 'collapses' to a straight line at perfect tuning. Two global indicators aide the iterative tuning: the Phase Indicator (PI) approaches zero while the Modal Indicator Function (MIF) approaches unity as shape-tuning is optimized.*

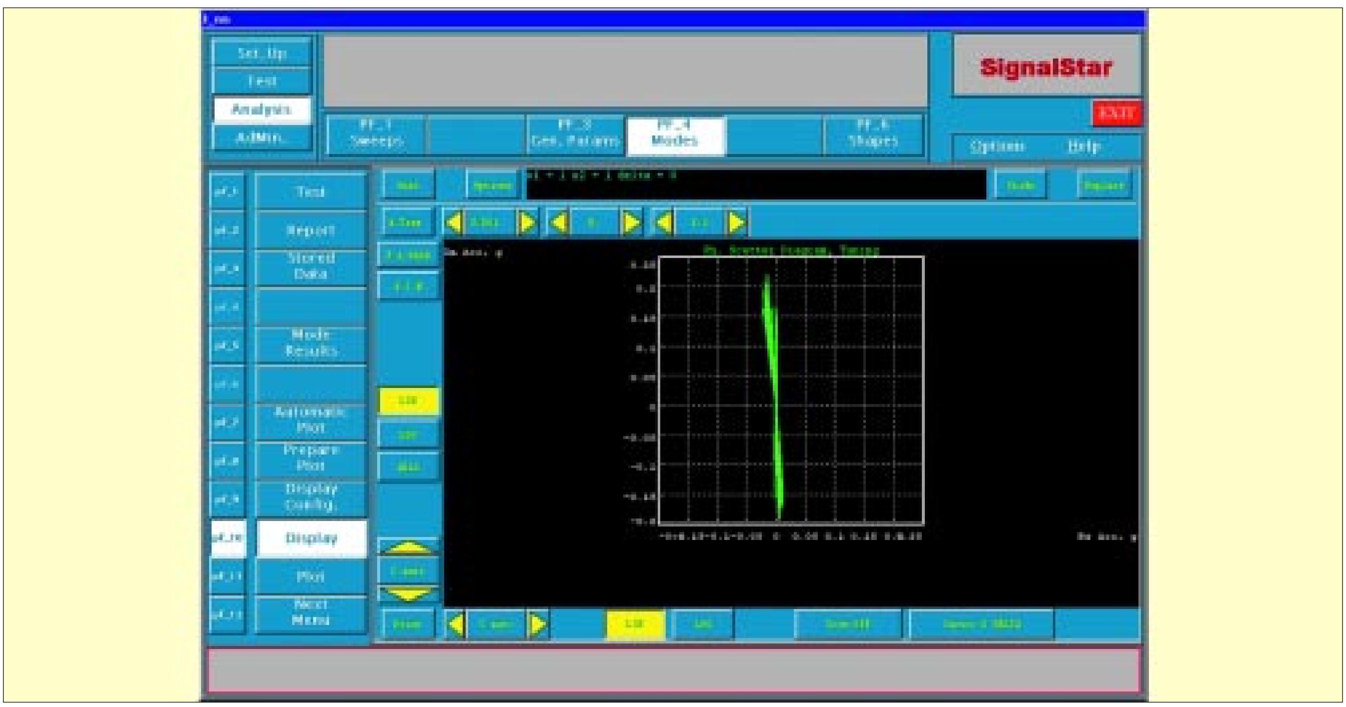


*Figure 4. A Phase Scatter diagram is formed by ploting the quadrature components of all response accelerations at the tuned frequency. Since the accelerations must be in phase quadrature with the applied forces, the real values (horizontal axis) approach zero and a vertical line along the imaginary axis results.*

surement of frequency. The force appropriation stage focuses on each mode individually, fine tuning to optimize the force and phase pattern of excitation and applying extreme frequency resolution to precisely tune and measure the structural characteristics.

This type of testing implies a large number of response channels; generally anything from 16 to thousands of accelerometers can be used at response measurement points. It is important that all channels are acquired, processed and displayed simultaneously. A high density but very informative display such as the multi Lissajous function (Figure 3) provides an indication of all channels to show the status of the phase between channels. The system calculates the Modal Indicator Function (MIF) and the Phase Indicator (PI) in real-time with each tuning iteration, for each measurement. Alternative displays such as the Phase Scatter diagram of Figure 4 and the animated mode shape of Figure 5 provide additional insight, each from a different physical perspective.

The appropriation may be performed as a manual operation, making adjustments to the force distribution and frequency until all parametric indicators are optimized. Alternatively, an automatic appropriation algorithm called the *Anderson* method may be employed. This algorithm automatically determines the force pattern for each mode to be isolated for modal extraction.

## Mathematical Basis of Force-Apportioned Normal Mode Testing

The force-motion relationships between $N$ degrees-of-freedom (DOF) within a structure can be well approximated by $N$ second-order linear differential equations with constant coefficients. These may be written efficiently in matrix format, typified by Equation 1.

$$[\mathbf{M}]\{\ddot{\mathbf{x}}\}+[\mathbf{C}]\{\dot{\mathbf{x}}\}+[\mathbf{K}]\{\mathbf{x}\}=\{\mathbf{F}\} \quad (1)$$

In Equation 1, the displacement motions are represented by the ($N$ element) vector $\mathbf{x}$. A dot over this vector indicates differentiation with respect to time, yielding velocity while two dots denote double-differentiation yielding acceleration. Forces applied to these same degrees-of-freedom are represented by the vector $\mathbf{F}$.

The $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ matrices represent mass, damping and stiffness respectively. In general, these matrices have non-zero off-diagonal elements so that the $N$ equations are coupled. The purpose of a Normal Mode test is to *uncouple* these equations. This is accomplished by identifying the *mode shapes*.

In general, a system with $N$ degrees-of-freedom can exhibit $N$ mode shapes. Each of these is a specific pattern of motions and each can be represented by a vector, $\boldsymbol{\phi}_n$. Normal Mode testing assumes the mode shapes will be real-valued. That is, every element in a $\boldsymbol{\phi}_n$ vector will be a real number, indicating that all degrees-of-freedom must move *in phase* or in exact phase opposition.

A *coordinate transformation* matrix is written by using each Normal Mode solution vector $\boldsymbol{\phi}_n$, as a single column. This transformation simply 'explains' any arbitrary pattern of motion, $\mathbf{x}$ as a linear combination of the $N$ Normal Mode shapes $\boldsymbol{\phi}_n$. The weighting coefficients of this summation are provided by the *modal participation vecto*r $\mathbf{q}$ as asserted in Equation 2.

$$\{\mathbf{x}\}=\left[\{\phi\}_1\cdots\{\phi\}_n\cdots\{\phi\}_N\right]\{\mathbf{q}\}=[\phi]\{\mathbf{q}\} \quad (2)$$

The Normal Mode solution vectors have an important mathematical property termed *generalized orthogonality* with respect to the mass, stiffness and damping matrices. This property means that the coordinate transformation matrix can be used to diagonalize all three matrices when applied as a *congruence transformation*. Diagonalizing the matrices amounts to uncoupling (solving) the equations of motion.

A congruence transformation is performed upon a matrix by *pre-multiplying* the matrix by the *transpose* of a coordinate transformation (the transformation written with rows replacing columns) and then *post-multiplying* by the transformation. When the transformation has generalized orthogonality with respect to the matrix, the result is a matrix with zero valued elements everywhere but on the diagonal.

$$[\phi]^T[\mathbf{M}][\phi]=[\diagdown\,\mathbf{d}_n\diagdown] \quad (3)$$

Equation 3 illustrates the congruence transformation of the mass matrix $\mathbf{M}$. It also illustrates a further point. Each Normal Mode vector $\boldsymbol{\phi}_n$, represents a shape, not an 'absolute' measure of displacement. The information content is the relative amplitude of any DOF compared to all of the other DOFs within a given mode shape. The value of each diagonal element, $\mathbf{d}_n$, is affected by the *length* of the corresponding $\boldsymbol{\phi}_n$ vector, as well as the shape it conveys.

Hence, we must *normalize* the mode shape vectors by specifying their length (i.e. the size of the largest element). For reasons that will become immediately evident, we will choose the length of each $\boldsymbol{\phi}_n$ so that its corresponding $\mathbf{d}_n$ is equal to 1. Vectors scaled in this manner are said to be *orthonormalized* with respect to mass. If we substitute equation 2 in Equation 1 and then pre-multiply both sides of the equation by the transpose of the coordinate transformation $\boldsymbol{\phi}$ the mass, stiffness and damping matrices are all subjected to congruence transformation as shown in equation 4.

$$[\phi]^T[\mathbf{M}][\phi]\{\ddot{\mathbf{q}}\}+[\phi]^T[\mathbf{C}][\phi]\{\dot{\mathbf{q}}\}+[\phi]^T[\mathbf{K}][\phi]\{\mathbf{q}\}=[\phi]^T\{\mathbf{F}\} \quad (4)$$

If the vector elements in $\boldsymbol{\phi}$ are *orthonormalized* vectors, the equations are simplified to the results of Equation 5. The mass matrix is reduced to an *identity* matrix with ones on the diagonal and zeros elsewhere. The stiffness matrix becomes diagonal with the square of the natural frequency (in radians-per-second) of each mode on the diagonal. The damping matrix collapses to a diagonal matrix with central elements equal to twice the viscous damping factor times the natural frequency for each mode.

$$[\diagdown\,\mathbf{1}\diagdown]\{\ddot{\mathbf{q}}\}+[\diagdown\,\mathbf{2}\xi_n\omega_n\diagdown]\{\dot{\mathbf{q}}\}+\left[\diagdown\,\omega_n^2\diagdown\right]\{\mathbf{q}\}=[\phi]^T\{\mathbf{F}\}=\{\mathbf{Q}\} \quad (5)$$

Hence the *only* coupling between the resulting $N$ equations is in the right-hand side. That is, the manner in which the structure is forced determines the coupling between the Normal Modes in Equation 5. Note that the excitation of these uncoupled equations is not the *physical* force vector $\mathbf{F}$ (directly), it is the *generalized force* vector, $\mathbf{Q} = \boldsymbol{\phi}^T\,\mathbf{F}$.

Consider what happens when we choose the physical excitation force vector $\mathbf{F}$ in accordance with Equation 6. That is, we excite the structure with an array of sinusoidal forces all at the *same* frequency with amplitude distributions proportional to *one* of the Normal Mode shapes (weighted by the mass matrix).

$$\{\mathbf{F}\}=\alpha[\mathbf{M}]\{\phi\}_n \quad (6)$$

This results in a *generalized force* vector $\mathbf{Q}$ with zero values for all elements save the one corresponding to the selected Normal Mode shape as shown in Equation 7. That is, only *one mode* is excited *regardless of the frequency*. The decoupling of the equations is now complete and each mode may be studied in isolation of the others.

$$\{\mathbf{Q}\}=\alpha[\phi]^T[\mathbf{M}]\{\phi\}_n=\begin{Bmatrix}\mathbf{0}\\ \vdots\\ \alpha\\ \vdots\\ \mathbf{0}\end{Bmatrix} \quad (7)$$

Hence the purpose of *Apportioned-Force* Normal Mode testing is to iteratively tune the *distribution of applied forces* until the generalized force is nulled for all but the mode sought. While the process is imperfect due to the finite number of DOFs excited, it can provide mode shapes of the highest accuracy when applied to structures characterized by (predominantly) real modes.

The sets of forces are first initialized from the FRFs measured during the identification stage and then adjusted iteratively by an automatic process. Controls are provided to stop iteration when the appropriation is correct.

### Mode Shape Acquisition

Once a mode has been apportioned, its shape may be 'harvested' by recording the response amplitudes of all accelerometers while maintaining the structure in a brief resonant dwell. Since the apportioned excitation maintains motion in a single isolated mode, there is no need to apply any form of mathematical shape separation.

### Generalized Parameters

The generalized mass, stiffness and damping of each mode can be evaluated from the total driving power required to maintain the structure in resonant dwell. The modal mass can also be independently evaluated by an alternative test that simulates a structural change at the driving degrees-of-freedom. These calculations are termed the Complex Power and Quadra-

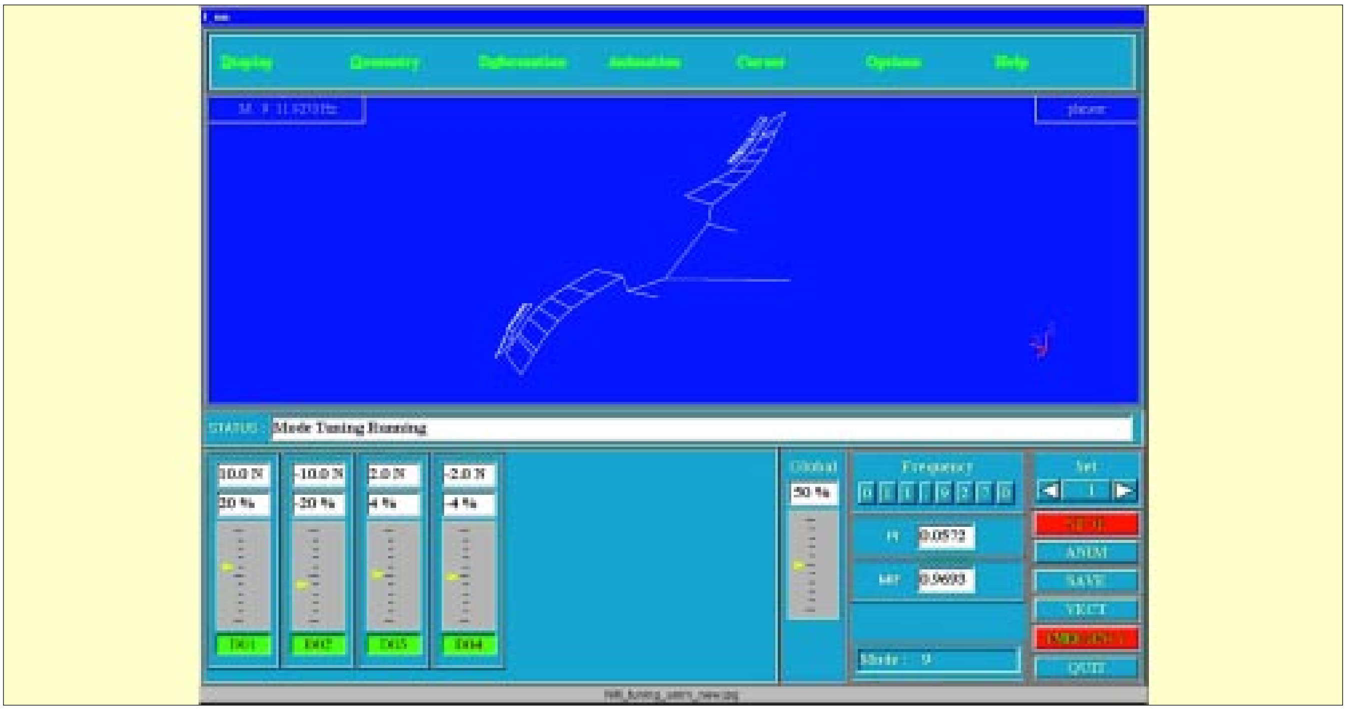


*Figure 5. An animated mode-shape display made during Appropriation tuning illustrates the character of the mode and the quality of its isolation and capture. Properly isolated modes are free of the 'galloping nodes' frequently encountered in global excitation tests. Appropriate symmetry and continuity may be verified at a glance.*


*Figure 6. A Complex Power function is formed for each mode by summing all drive-site force-velocity products during a narrow frequency sweep about each resonance. Ideally, the real or active component (green) exhibits a maximum at the undamped natural frequency while the imaginary or reactive component crosses zero at this same point.*

ture Force methods, respectively.

The Complex Power function (Figure 6) is formed by summing the real and imaginary components of the Force x Velocity product at all shaker locations. This is done using the appropriated force distribution for a specific mode. The Complex Power is evaluated over a narrow frequency range centered on the subject resonance. This results in a complex spectrum that is subsequently curve-fit (Figure 7) with a single degree-of-freedom analytic model, yielding the generalized mass, stiffness and damping.

Quadrature Force testing (Figure 8) provides an alternative measure of the generalized mass of a mode. In this procedure, the structure is held in force-appropriated resonant dwell and a second set of (small) forces is introduced at the drive sites (by the same shakers). These secondary forces are proportional in amplitude to the basic appropriated forces, but are in phase quadrature with them. The amplitude (and sense) of each quadrature force is equal to an adjustable quadrature coefficient multiplied by the corresponding appropriated force. Several tests are run, each with a different quadrature coefficient (spanning ± several percent).

The addition of quadrature forces 'detunes' all indications of proper force appropriation. To correct this, the driving *frequency* is slightly changed until indication of proper appro-

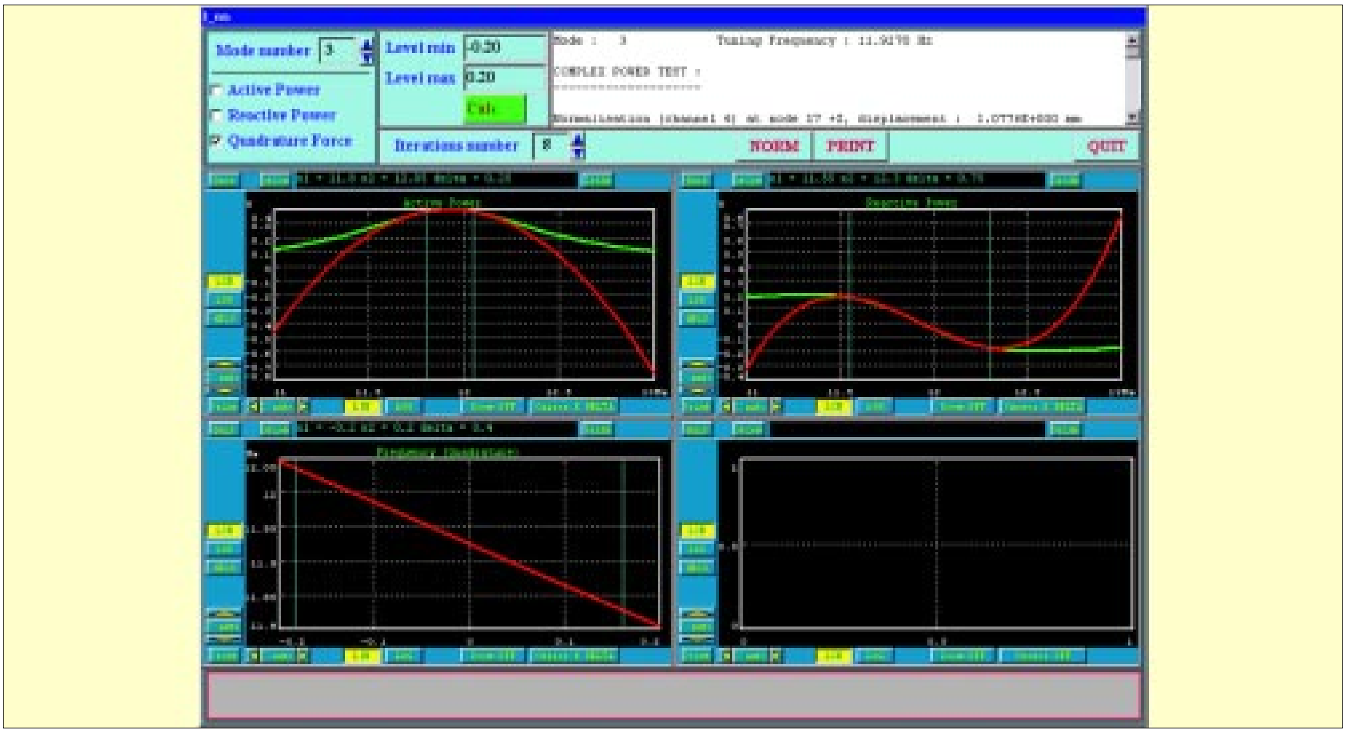


*Figure 7 .Curve-fits to the measured Complex Power and Quadrature Force functions provide two independent measurements of Modal Mass plus Modal Stiffness and Damping.*

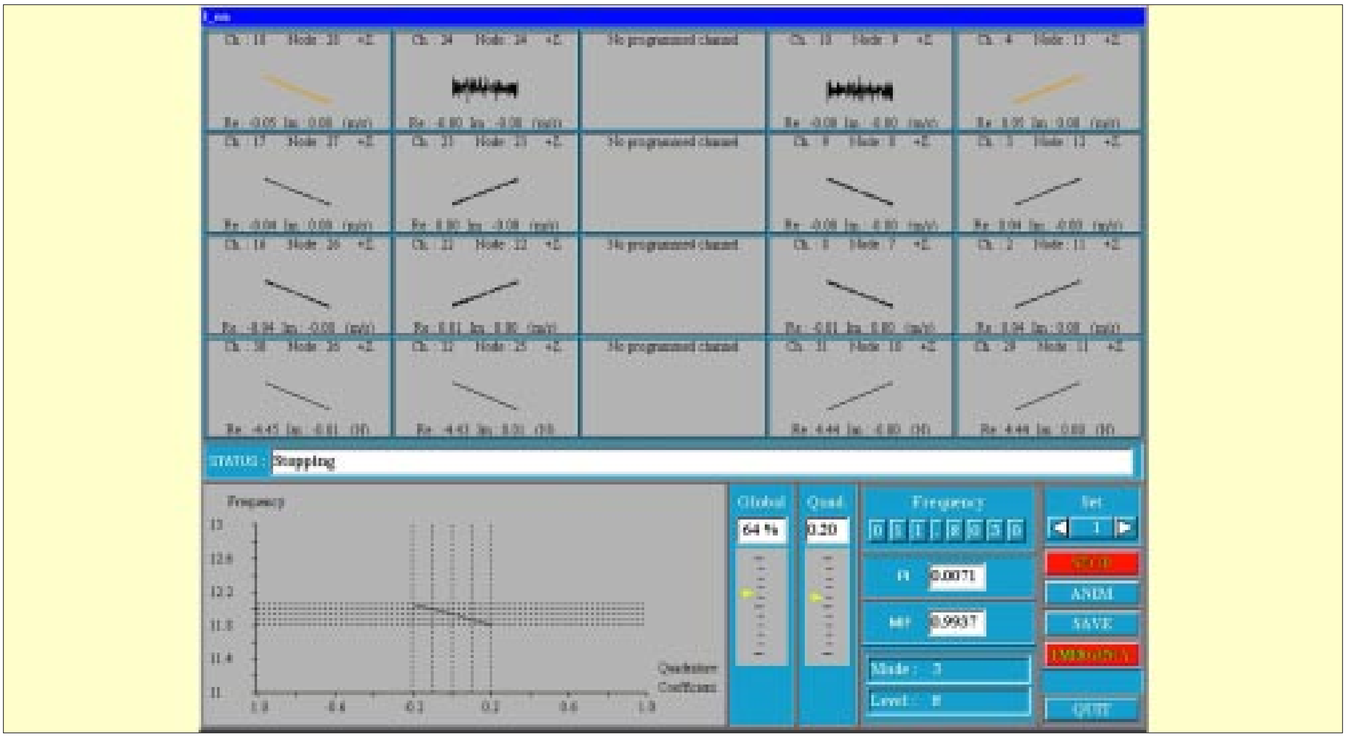


*Figure 8. A Force-in-Quadrature test applies small aditional forces in quadrature with and proportional to the forces necessary to optimize the apportioned tuning. The sign and magnitude of these additional forces is stepped over a percentage of the primary drive and at each step the frequency is re-tuned to optimize the appropriation. A linear function of frequency-versus-quadrature coefficient results (lower-left).*

priation is reachieved. A plot of frequency-versus-Quadrature Coefficient results. This function is a straight line (for an ideal SDOF system) and the slope of the line is determined by the generalized mass. A simple curve-fit is performed and the generalized mass is determined from the resulting slope.

## Linearity Check

It is well understood that real aerospace structures can and do exhibit amplitude non-linearity. One of the merits of force-appropriated Normal Mode testing is that it allows the linearity of each mode to be examined and characterized. This is accomplished by conducting a series of force appropriations, each using a different global force level. A plot of the resulting natural frequency-versus-force quickly identifies 'soft' or 'hard' nonlinear response. Figure 9 illustrates such characterization from a very linear structure: an 11.9 Hz mode shows (extremely slight) 'soft' response, decreasing in frequency by 12 mHz over a 5:1 span of global force level.

## Conclusions

The dynamic properties of a structure are naturally segregated by frequency and spatial orthogonality. By respecting

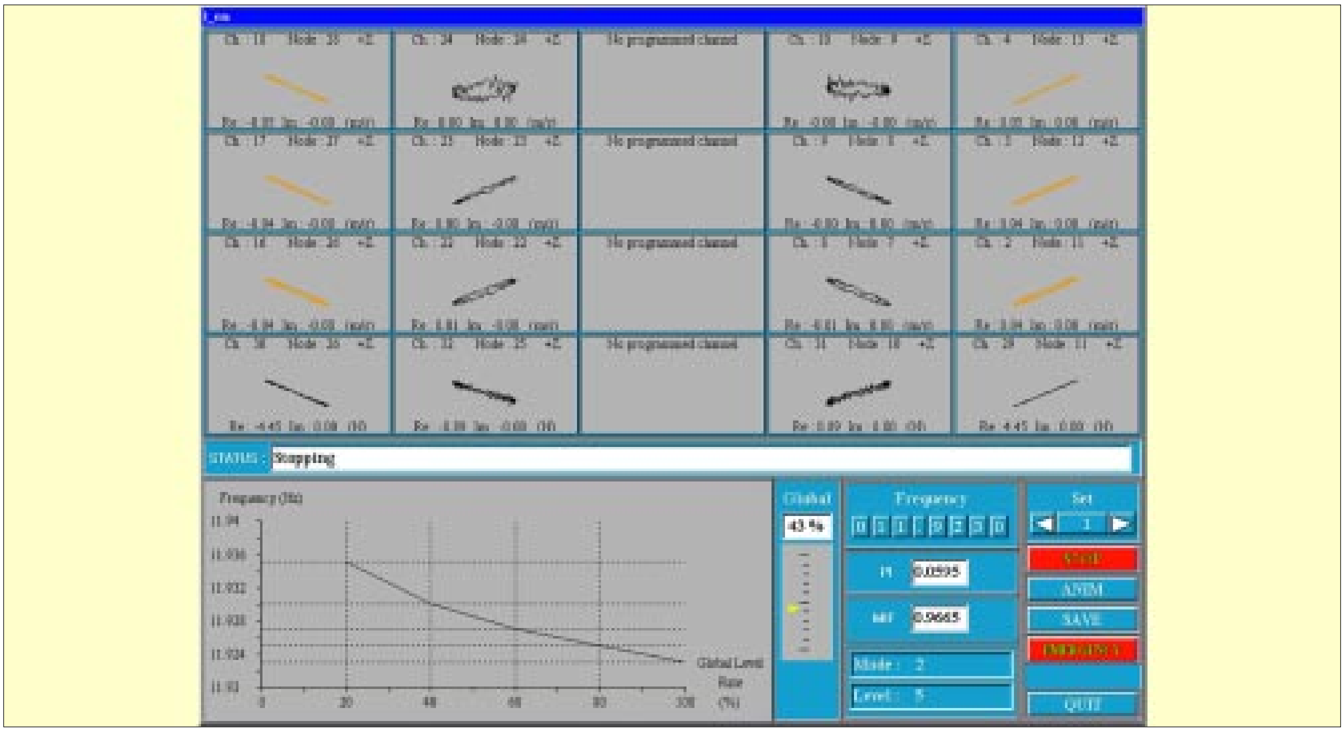

*Figure 9. System linearity with force is investigated on a mode-by-mode basis. The global forcing level is stepped across a desired range and the frequency is adjusted to optimize tuning at each level. This results in a plot of natural frequency-versus-force level as shown at lower-left.*

both of these characteristics, an appropriated force Normal Mode test can identify the natural parameters of each mode in isolation. In particular, this type of analysis provides extremely precise mode shapes even when adjacent modes overlap in modal bandwidth. This results in a modal model of the highest integrity and precision, one in which the analyst can have the highest confidence. Such testing permits personal interaction with the structure under test and affords the opportunity to evaluate structural linearity.

Sinusoidal testing provides a unique opportunity for the tester to interact with the test specimen on a mode-by-mode basis. The structure can be explored with eye, ear and hand during the examination. This "hands on the structure" approach is key to gaining personal insight of the structure's behavior. Broadband techniques do not lend themselves to such interaction and thus fail to capitalize upon intuitive intellect.

Normal mode testing has been used for many years in the Aerospace industry and in those years two key components, the computing platform and the application software, have evolved considerably. The process of optimizing the phase and force pattern for each mode involves intensive processing, which would previously have required extremely expensive computers. Today a standard, dual CPU Pentium PC can perform the task with ease. The application software such as the Data Physics Matrix system has seen continuous refinement, resulting in shortened test times and advanced graphical user interfaces, providing ease of use.

Sinusoidal testing suffers all of the mechanical fixturing liabilities of random testing. However, its narrow-band nature allows testing large structures with smaller shakers and amplifiers than random testing permits. This means less expensive, more transportable excitation subsystems can service a broader range of structures and that shaker fixturing is less influential and onerous. More importantly, force-appropriated sinusoidal normal mode measurement is possible when the structure is heavilly damped and even when it exhibits nonlinear behavior. Controlled-sine excitation permits exciting these elusive modes at reasonable energy levels without imparting local over-stressing. In a broader sense, a Normal Mode test is recognized as a sub-set of the many types of MDOF forced/motion tests that may be employed to validate a complex FEM modeling of a modern composite structure.

It is understood that some structures will have modal responses that cannot be isolated using appropriation methods. The reason might be that it is physically impossible to gain access to excite at the correct positions or that the damping distribution is such that complex rather than normal modes must be extracted for a proper model. In this case, conventional global excitation modal analysis techniques using curve-fitting may provide the right answer. In other situations, it may prove more appropriate to validate the design model using alternative correlation strategies independent of modal characteristics. These typically involve performing a sequence of prescribed forced-motion tests that may be of sine, random or transient nature.

Today, a total FEM validation tool must perform all types of modal testing, from the simplest impulse survey to a MIMO (Multiple Input Multiple Output) random test to evaluate complex modes to a force-appropriated Normal Mode test. It must also be capable of performing a broad range of forced-motion tests, imparting controlled forces of various wave types to multiple shakers simultaneously. The system must also deal with a massive response-channel count to fully characterize large structures during a single instance of excitation. SV

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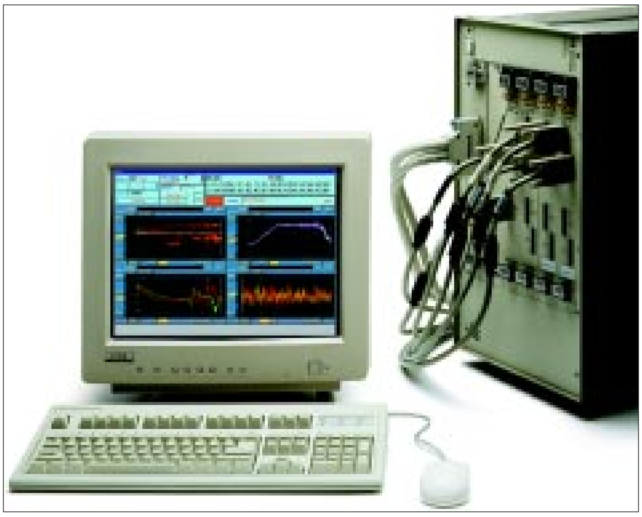
## Data Physics SignalStar Matrix



Measurements presented in this article were made with the Data Physics SignalStar® Matrix system shown above. This modern shaker controller/analyzer utilizes industry-standard Agilent VXI modular hardware and provides a complete solution to all high-end shaker test requirements.

In addition to performing force-appropriated Normal Mode testing, Matrix can run all classical shake tests including Random, Sine, Shock, SRS, Mixed Modes and Road Simulation. SignalStar Matrix not only delivers unparalleled performance for every type of single axis shaker test, it also delivers the ability to control up to 16 shakers in multiaxis shaker tests, working equally well with hydraulic and electrodynamic shakers.

The high-performance technical specifications and intuitive user interface enhance industry-standard vibration testing in all modes. Complete post-processing analysis and extensive report generation is provided. A broad range of hardware/software options support other dynamic measurement activities including random MIMO testing, modal curve-fitting and display, order-tracking for rotating machinery analysis and octave- and 1/3-octave analysis for acoustic studies.

More information is available at www.dataphysics.com.